Judiciuos Guessing

1. Solving ODE's of the form
$$ay'' + by' + cy = p(t)$$
where $a, b, c \in \mathbb{R}$ and $p(t)$ is a polynomial of degree n.

   (a) If $c \neq 0$, initial guess is
$$\psi(t) = A_n t^n + A_{n-1} t^{n-1} + \cdots + A_1 t + A_0$$

   (b) If $c = 0$ and $b \neq 0$, initial guess is
$$\psi(t) = \left(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_1 t + A_0\right) t$$

   (c) If $c = b = 0$, initial guess is
$$\psi(t) = \left(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_1 t + A_0\right) t^2$$

2. Solving ODE's of the form
$$ay'' + by' + cy = p(t)e^{\alpha t}$$
where $a, b, c, \alpha \in \mathbb{R}$ and $p(t)$ is a polynomial of degree n.

   (a) If α is *not* a root of the characteristic polynomial, initial guess is
$$\psi(t) = \left(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_1 t + A_0\right) e^{\alpha t}$$

   (b) If α *is* a root of the characteristic polynomial with multiplicity $m_\alpha = 1$, initial guess is
$$\psi(t) = \left(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_1 t + A_0\right) te^{\alpha t}$$

   (c) If α is a root of the characteristic polynomial with multiplicity $m_\alpha = 2$, initial guess is
$$\psi(t) = \left(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_1 t + A_0\right) t^2 e^{\alpha t}$$

3. Solving ODE's of the form
$$ay'' + by' + cy = p(t)e^{\alpha t}\cos\beta t$$
or
$$ay'' + by' + cy = p(t)e^{\alpha t}\sin\beta t$$
where $a, b, c, \alpha, \beta \in \mathbb{R}$ and $p(t)$ is a polynomial of degree n.

   (a) If $\alpha + i\beta$ is *not* a root of the characteristic polynomial, initial guess is
$$\Psi(t) = (A_n t^n + \cdots + A_1 t + A_0)\, e^{\alpha t}\cos\beta t + (B_n t^n + \cdots + B_1 t + B_0)\, e^{\alpha t}\sin\beta t$$

   (b) If $\alpha + i\beta$ *is* a root of the characteristic polynomial, initial guess is
$$\Psi(t) = t\left[(A_n t^n + \cdots + A_1 t + A_0)\, e^{\alpha t}\cos\beta t + (B_n t^n + \cdots + B_1 t + B_0)\, e^{\alpha t}\sin\beta t\right]$$