Nonlinear Systems

SIR Model

John Meth

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$S(t)$ is the number of susceptible individuals

$I(t)$ is the number of infected individuals

$R(t)$ is the number of recovered individuals

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A linear system in S, I, and R would look like

$$\begin{aligned}
\frac{d}{dt}S &= a_1 S + a_2 I + a_3 R \\
\frac{d}{dt}I &= b_1 S + b_2 I + b_3 R \\
\frac{d}{dt}R &= c_1 S + c_2 I + c_3 R
\end{aligned}$$

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Our model of disease spread in S, I, and R is *nonlinear*

$$
\begin{aligned}
\frac{d}{dt}S &= -\alpha S I \\
\frac{d}{dt}I &= \alpha S I - \gamma I \\
\frac{d}{dt}R &= \gamma I
\end{aligned}
$$

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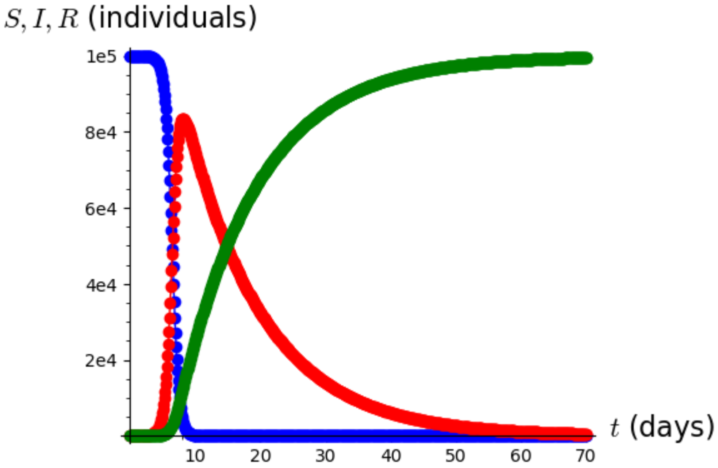
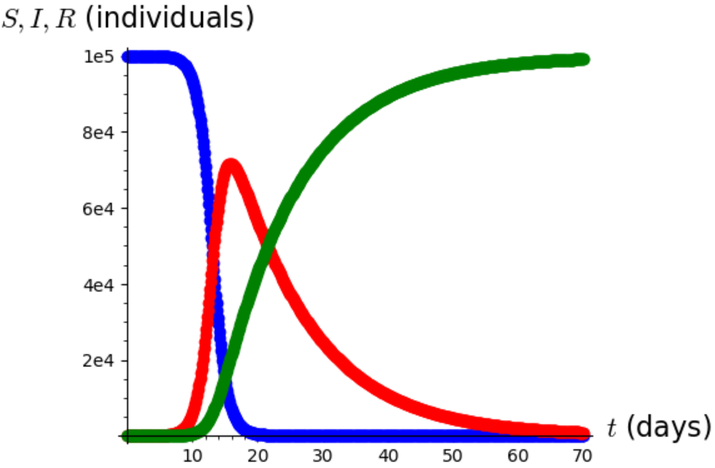
Our model of disease spread in S, I, and R is *nonlinear*

$$\begin{aligned}\frac{d}{dt}S &= -\alpha S I\\ \frac{d}{dt}I &= \alpha S I - \gamma I\\ \frac{d}{dt}R &= \gamma I\end{aligned}$$

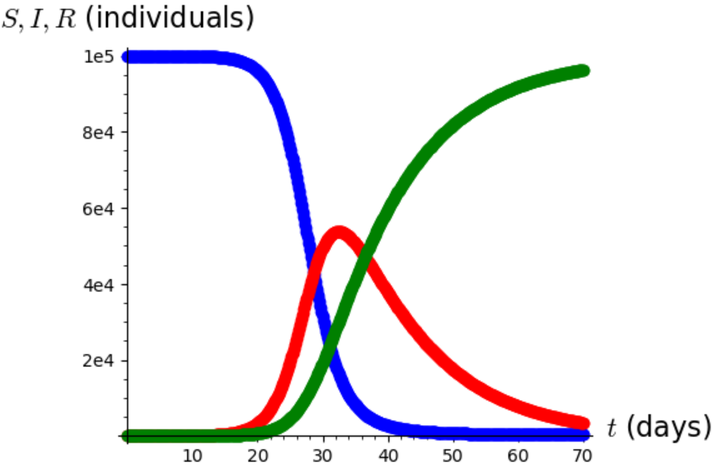
$$\alpha = \left\{\begin{array}{l}\text{Proportion of possible}\\ \text{contacts that actually}\\ \text{occur}\end{array}\right\} \cdot \left\{\begin{array}{l}\text{Proportion of those}\\ \text{contacts that actually}\\ \text{result in an infection}\end{array}\right\}$$



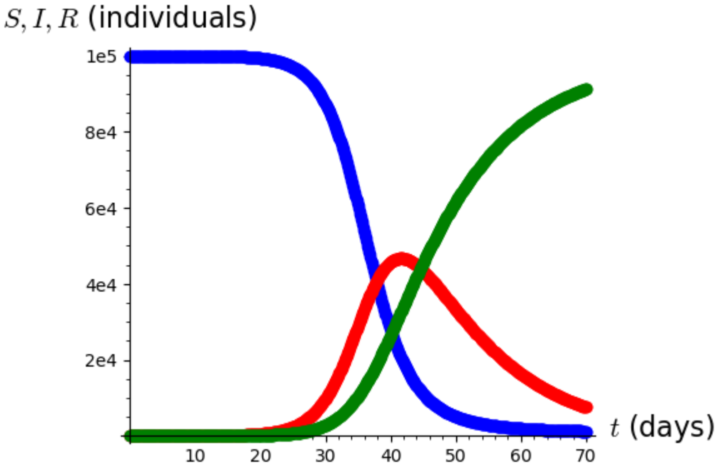
$\alpha = .00002$



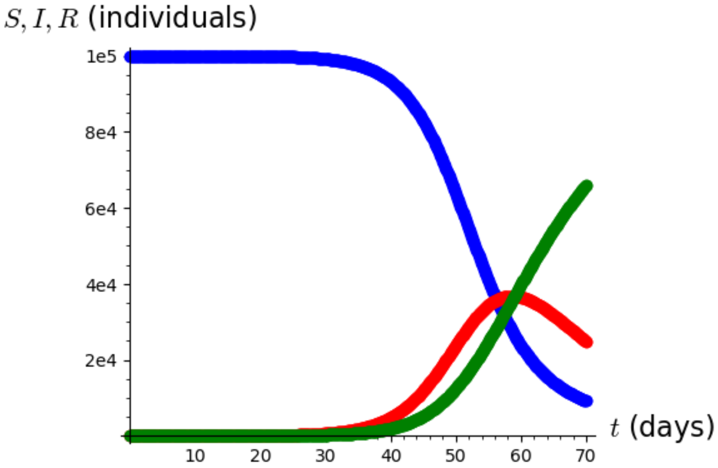
$\alpha = .00001$



$\alpha = .000005$



$\alpha = .000004$



$\alpha = .000003$

- https://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology
- http://math.colorado.edu/~stade/CLS/sage3.html
- https://cocalc.com/
- http://faculty.sfasu.edu/judsontw/ode/html-20190821/systems01.html
  Section 2.1.3 of Judson